ANALYTICAL METHOD FOR SUPPRESSING GIBBS LOBES IN SPECTRAL ANALYSIS

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ABSTRACT

The method of suppression of side lobes or Gibbs pulsations arising during the Fourier transform is considered. The method consists in calculating an analytical spectrum representing the convolution of a sinusoidal signal and a rectangular window function and then subtracting it from the experimental spectrum in the region where the side lobes are located. The spectral resolution of the proposed method corresponds to a rectangular window, and the suppression of side lobes exceeds 40 dB. Examples of using this method for variations in solar activity, carbon dioxide, total ozone, and comparison with some traditional methods are given.

1. INTRODUCTION

Fourier transform (FT) is widely used in signal processing and time series analysis in various fields of geophysics. One of the problems associated with the use of FT is the problem of suppressing side lobes or Gibbs pulsations. For a classical FT with a rectangular window, the amplitude of the first side lobe is 13 dB. The appearance of lateral pulsations occurs due to the fact that in the frequency domain there is a convolution of the signal function with a rectangular window function [1]. To attenuate such pulsations, filtration methods have been developed, which consist in replacing a rectangular window with other windows [2]. The use of windows other than rectangular windows leads along with weakening of side lobes, to broadening of the main peak and to a worse spectral resolution. Therefore, the choice of the window function is usually associated with a trade-off between the degree of attenuation of side lobes and the width of the main peak. Replacement of the rectangular window function with others, sometimes very complex functions remains common to most of the methods developed in recent decades, for example [3-5].

2. METHOD

We have proposed an alternative method for suppressing Gibbs lobes, which consists in using analytical expressions of FT for signals of various shapes. The essence of the method is to subtract the analytical spectrum obtained in the region where the side lobes are located from the experimental spectrum. Consider a series $\varphi(t)$ containing a sine harmonic with a period $t1$, frequency $f1 = 1/t1$, amplitude A, length N. For a rectangular pulse with amplitude A and width N located on the positive axis, the analytical expression for the spectral power $P(f)$ has the form (1).

$$P(f) = (A\,N)^2 \sin^2(\pi f N)/(\pi f N)^2 \qquad (1)$$

To construct an analytical expression for the convolution of signal and rectangular window, we use, along with the expression for the direct Fourier transform (2), the Euler relation (3), the analytical expression for the Fourier transform for the *unlimited* sine (4) and the FT property of the phase shift of when the function $\varphi(t)$ is shifted by the argument (5).

$$\Phi(f) = \int_{-\infty}^{\infty} \varphi(t)\exp(-j\,2\pi f t)\,dt \qquad (2)$$

$$\exp(j\,2\pi f_0 t) = \cos(j\,2\pi f_0 t) + j\sin(j\,2\pi f_0 t) \qquad (3)$$

$$\sin(2\pi f_0 t) = (\delta(f-f_0) + \delta(f+f_0))/2j \qquad (4)$$

$$\varphi(t+t_0) = \Phi(f)\exp(-j\,2\pi f t) \qquad (5)$$

After converting, we obtain expression (6) for the spectral power, and expression (7) for the spectral amplitude of a *limited* sine signal located on the positive axis. Expression for the cosine spectral power can be found in a similar way (8).

$$P(f) = \left(\frac{A}{2}\right)^2 \left(\frac{Sin\,(2\pi(f-f_0))N}{(2\pi(f-f_0))} - \frac{Sin\,(2\pi(f+f_0))N}{(2\pi(f+f_0))}\right)^2 + \left(\frac{A}{2}\right)^2 \left(\frac{Sin^2\,(\pi(f-f_0))N}{(\pi(f-f_0))} - \frac{Sin^2\,(\pi(f+f_0))N}{(\pi(f+f_0))}\right)^2 \qquad (6$$

$$A(f) = 2/N\,\sqrt{P(f)} \qquad (7)$$

$$P(f) = \left(\frac{A}{2}\right)^2 \left(\frac{Sin\,(2\pi(f-f_0))N}{(2\pi(f-f_0))} + \frac{Sin\,(2\pi(f+f_0))N}{(2\pi(f+f_0))}\right)^2 + \left(\frac{A}{2}\right)^2 \left(\frac{Sin^2\,(\pi(f-f_0))N}{(\pi(f-f_0))} + \frac{Sin^2\,(\pi(f+f_0))N}{(\pi(f+f_0))}\right)^2 \qquad (8)$$

The frequency of the main peak f_0 (or several peaks f_{0i}) is determined directly from the experimental spectrum. To exclude side lobes, the analytical spectrum (6) in the side lobes region $f_1 \geq f \geq f_2$ should be subtracted from the experimental spectrum. Frequencies of the first minima of main peak are $f_1 = f_0 - 1/N$, $f_2 = f0 + 1/N$.

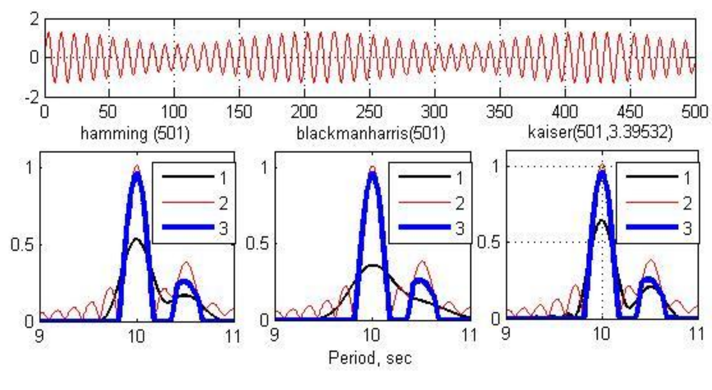

Figure 1. Comparison with traditional windowing methods. The upper figure shows the sum of two sinusoids with amplitudes of 1.0 and 0.3 and periods of 10 and 10.5 sec. The lower figure shows the spectra. 1 - windowing methods, 2 - classical FT, 3 - analytical method

EXAMPLES

Figure 1 illustrates a comparison of the analytical method with the traditional window methods of Hamming, Blackman-Harris and Kaiser (with the parameter a = 0.39). A time series consisting of the sum of two sine waves with close periods was used for comparison.

The proposed method has the maximum possible spectral resolution corresponding to a rectangular window and a spectral gain close to unity (in this example, 0.96). The degree of suppression of the nearest side lobes is more than 100 times or 40 dB. The high resolution of the method allows the analysis of time series containing two or more oscillations with close frequencies.

This is especially important in the case of the presence of a powerful fundamental harmonic, for example, the annual variation in geophysical processes. In the case of the time series under consideration, it should be noted that the second weak peak was confidently distinguished by the analytical method. It should be noted that the degree of suppression of side lobes depends on the accuracy of determining the frequency of the fundamental harmonic f_0. When calculating the experimental spectrum using formula (2), the frequency step in the region of the maximum of the fundamental harmonic should be sufficiently small. This is due to the fact that the spectral interval occupied by the main peak, i.e. the region of the spectrum where pulsation is not suppressed is determined by the value $f_0 \pm 1/N$.

Figure 2 shows examples of using the analytical method for three geophysical time series from [6-8]. In variations of solar activity, four fundamental harmonics are distinguished with periods of 10.1, 10.5, 11.0, and 11.9 years, and a weaker harmonic of 13.1 years. A feature of closely spaced first 3 harmonics is the coincidence of the maxima of the main peaks with the maxima of side lobes. The calculation by the analytical method showed that the amplitude of the spectral peak with a period of 10.5 years is noticeably lower than for the classical FT.

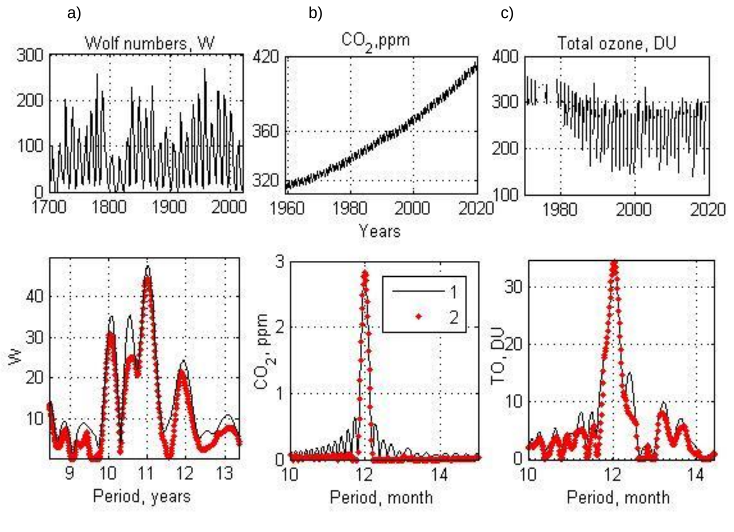

Figure 2. Time series and spectra of a) - Wolf numbers, b) - carbon dioxide concentrations at Mauna Loa, c)- zonal-average total ozone (latitudinal zone 75° S -80° S).
1 - classical FT, 2 - analytical method

In general, for the analyzed period, the spectral maxima of solar activity variations in the 11-year region tend to a periodicity of one year.

The spectrum of CO_2 variations at st. Mauna Loa in the short-period region (< 2 years) shows only annual and semiannual harmonics. This is probably due to the measurement data processing procedure and the high degree of smoothing of the primary measurement data. All peaks located near the annual and semiannual harmonics are due to sidelobes.

Satellite measurements of the total ozone in the latitudinal zone of 75°-80° S have a gap in 1976-1979 and irregular skips during the polar night. According to the classical FT, along with the annual harmonic, the oscillation with a period of about 12.4 months has a noticeable amplitude. As can be seen from the figure, this oscillation does not manifest itself in the spectrum with suppressed sidelobe pulsations. At the same time, the asymmetry of the main peak in the filtered spectrum and noticeable pulsations in the short-period region should be noted. These features of the spectrum are possibly related to the influence of the 1976-1979 discontinuity and the broadening of the central peak due to intra-annual gaps, the filling of which with measurement data was unstable and varied in 1979-2018 from 40 to 60%.

CONCLUSION

1. An analytical method for suppressing Gibbs pulsations in the practice of spectral analysis is proposed.
2. The proposed method has the maximum possible spectral resolution corresponding to a rectangular window.
3. A spectral gain of the method is close to unity.
4. Out-of-band sidelobe suppression is greater than 40 dB.
5. The use of the method makes it possible to assess the reliability of closely spaced spectral peaks observed in the spectrum and the appearance of false peaks due to gaps in measurements.

REFERENCES

1. ***Otnes R.K., Enochson L.*** **Applied Time Series Analysis. Vol. I. Basic techniques 1978.**
2. ***Harris F.J.*** **On the Use of Windows for Harmonic Analysis with the Discrete Fourier Transform -// IEEE, 1978. – vol. 66, P. 51-84.**
3. ***Chang J., Changhong Y. L. and Chen*** **Y. A new family windows- convolution windows and their application.** ***Sci China Ser E-Tech Sci*****, vol.48 no.4, pp. 468-480, 2005.**
4. ***Okarma K.*****: Polynomial windows with low sidelobes' level. Signal Processing, vol. 87, April 2007, pp. 782-788.**
5. ***Shriwastave S., Gupta A., Thakre V*****. Suggested a one variable window for improve in side lobe reduction in fir filter. //Int. Journal of Research and Analytical Reviews. 2020. V.7. P. 471-479.**
6. **Yearly sunspot numbers (http://sidc.oma.be/html/sunspot.html).**
7. **Monthly average Mauna Loa CO_2 (https://www.esrl.noaa.gov/gmd/ccgg/trends/).**
8. **SBUV Zonal mean total ozone data (https://acd-ext.gsfc.nasa.gov/Data_services/merged/).**